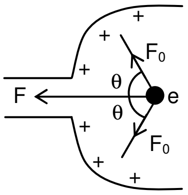
ANSWER OF MODEL PAPER FOR AIEEE

PHYSICS

1. **Ans.** Greater than v_0 **Reason:** When electron reaches near curved surface the force due to induces charges accelerates to electron.

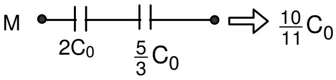


2. **Ans.** 18 μC. **Reason.** $I = \frac{12}{6+2} = \frac{3}{2}$

$V = 12 - 2 \times \frac{3}{2} = 9V$

$Q = CV = 2 \times 9 = 18\mu C.$

3. **Ans.** $\frac{10}{11}C_0$ **Reason:**



4. **Ans.** 320 cm **Reason:** In case of zero deflection in galvanometer.

$V_{AJ} = \frac{E}{2}$

$\therefore iR_{AJ} = \frac{E}{2}$ or $\left(\frac{E}{15r+r}\right)\left(\frac{15r}{600}\right)AJ = \frac{E}{2}$

Solving this equation, we get

AJ = 320 cm.

5. **Ans.** 2 IaB **Reason:** Initially F_1 = mg + IaB (down wards) when the direction of current is reversed

F_2 = mg – IaB (down wards) $\Rightarrow \Delta F = 2$ IaB.

6. **Ans.** $\frac{T}{2\sqrt{3}}$ **Reason :** $T = 2\pi\sqrt{\frac{I}{MH}}$; MI of each part $\frac{1}{6^3}$ and magnetic moment of each part = $\frac{M}{6}$. So net MI of system $= \frac{1}{6^3} \times 6 = \frac{1}{6^2}$ and net magnetic moment

$= \frac{4M}{6} - \frac{2M}{6} = \frac{M}{3}$.

$\therefore$ time period of the system

$T' = 2\pi\sqrt{\frac{I/36}{(M/3)H}} = \frac{1}{2\sqrt{3}}2\pi\sqrt{\frac{I}{MH}} = \frac{T}{2\sqrt{3}}$

7. **Ans.** 10 πmV **Reason :** the induced emf between centre and rim of the rotating disc is

$E = \frac{1}{2}B\omega R^2 = \frac{1}{2} \times 0.1 \times 2\pi \times 10 \times (0.1)^2 = 10\pi \times 10^{-3}$ volt.

8. **Ans.** 80 Hz **Reason:** With dc : $P = \frac{V^2}{R} \Rightarrow R = \frac{(10)^2}{20} = 5\Omega$

With ac : $P = \frac{V_{rms}^2 R}{Z^2} \Rightarrow Z^2 = \frac{(10)^2 \times 5}{10} = 50\ \Omega^2$

Also $Z^2 = R^2 + 4\pi^2 v^2 L^2$

$\Rightarrow 50 = (5)^2 + 4(3.14)^2 v^2 (10\times10^{-3})^2 \Rightarrow v = 80$ Hz.

9. **Ans.** The peaks at R and S would remain at the same wavelength and cut off wavelength at P would decrease **Reason:** Peak on the graph represent characteristic X-ray spectrum. Every peak has a certain wavelength, which depends upon the transition of electron inside the atom of the target. While $\lambda°_{min}$ depends upon the accelerating voltage (As $\lambda_{min} \propto 1/V$)

10. **Ans.** $\sqrt{\frac{8}{3}}$ **Reason:** de-Broglie wavelength $\lambda = \frac{h}{mv_{rms}}$, rms velocity of a gas particle at the given temperature (T) is given as $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT$

$\Rightarrow v_{rms} = \sqrt{\frac{3kT}{m}} \Rightarrow mv_{rms} = \sqrt{3mkT}$

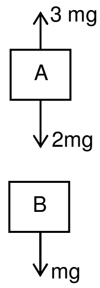
$\therefore \lambda = \frac{h}{mv_{rms}} = \frac{h}{\sqrt{3mkT}}$

$\Rightarrow$

$\frac{\lambda_H}{\lambda_{He}} = \sqrt{\frac{m_{He}T_{He}}{m_H T_H}} = \sqrt{\frac{4(273+127)}{2(273+27)}} = \sqrt{\frac{8}{3}}$

11. **Ans.** P.E increases and K.E decreases **Reason:** P.E $\propto -\frac{1}{r}$ and K.E $\propto \frac{1}{r}$. As r increases so K.E decreases but P.E increases.

12. **Ans.** g/2, g. **Reason:** After string is cut, free body diagram of block A gives :



2m a_A = 3mg – 2mg

or $a_A = \frac{mg}{2m} = \frac{g}{2}$

Free body diagram of block B gives :

$\therefore ma_B = mg$

or $a_B = g$.

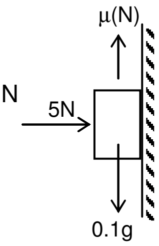
13. **Ans.** $\frac{2}{3}\frac{m\upsilon_0^2}{x_0^2}$. **Reason.** $V_A = V_0 \Rightarrow V = \frac{mV_0}{m+2m} = \frac{V_0}{3}$

$\Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}kx_0^2 + \frac{1}{2}3m\left(\frac{v_0}{3}\right)^2$

$\Rightarrow \frac{1}{2}kx_0^2 = \frac{1}{2}mv_0^2\left[1 - \frac{1}{3}\right]$

$\Rightarrow kx_0^2 = \frac{2}{3}mv_0^2 \Rightarrow k = \frac{2mv_0^2}{3x_0^2}$

14. **Ans.** 0.98 N. **Reason:** $\mu N = \frac{5}{10}(5) = 2.5$ N



0.1 g < 2.5 N

So $F_f = \frac{1}{10}(9.8) = .98$N.

15. **Ans.** 10,000 J **Reason:** WD = charge in PE = $P_2 - P_1$

P_2 = mg l/2 = 500 × 10 × 5/2 = 12500 J.

P_1 = 5 × m g a/2 = 5 × 100 × 10 × ½ = 2500

$\therefore$ WD = 12500 – 2500 = 10,000 J.

16. **Ans.** Sphere, disc, shell, ring. **Reason:** $I_{Sphere} < I_{Disc} < I_{Shell} < I_{Ring.}$ We know that body possess minimum moment of inertia will reach the bottom first and body possess maximum moment of inertia will reach the bottom of last.

17. **Ans.** $\sqrt{E} = \sqrt{E_1} + \sqrt{E_2}$.

Reason : $E_1 = \frac{1}{2}Kx^2 \Rightarrow x = \sqrt{\frac{2E_1}{K}}$, $E_2 = \frac{1}{2}Ky^2$

$\Rightarrow y = \sqrt{\frac{2E_2}{K}}$ and $E = \frac{1}{2}K(x+y)^2 \Rightarrow x + y = \sqrt{\frac{2E}{K}}$

$\Rightarrow \sqrt{\frac{2E_1}{K}} + \sqrt{\frac{2E_2}{K}} = \sqrt{\frac{2E}{K}} \Rightarrow \sqrt{E_1} + \sqrt{E_2} = \sqrt{E}$

18. **Ans**. 0.10 s **Reason:** For string, $\frac{\text{Mass}}{\text{length}} = m = \frac{10^{-2}}{0.4}$
$= 2.5 \times 10^{-2}$ kg/m
$\therefore$ velocity $v = \sqrt{\frac{T}{m}} = \sqrt{\frac{1.6}{2.5\times10^{-2}}}$ = 9 m/s
For constructive interference between successive pulses.
$\Delta t_{min} = \frac{2l}{v} = \frac{2(0.4)}{9} = 0.1$ sec.

19. **Ans**. 9 : 8. **Reason:** When source is approaching the observer, the frequency heard
$n_a = \left(\frac{v}{v - v_s}\right)\times n = \left(\frac{340}{340-20}\right)\times 1000 = 1063$ Hz
When source is receding, the frequency heard
$n_r = \left(\frac{v}{v + v_s}\right)\times n = \frac{340}{340+20}\times 1000 = 944$
$\Rightarrow n_a : n_r = 9 : 8$.

20. **Ans**. 15 cm, concave **Reason:** $\frac{1}{f} = \left(\frac{\mu_a}{\mu_b} - 1\right)\left[\frac{2}{R}\right]$
$\Rightarrow \frac{1}{f} = \left(\frac{1}{1.5} - 1\right)\times\frac{2}{10} \Rightarrow f = -15.$

21. **Ans.** 18 **Reason:** $n_1\lambda_1 = n_2\lambda_2$
$\Rightarrow n_2 = n_1 \times \frac{\lambda_1}{\lambda_2} = 12\times\frac{600}{400} = 18.$

22. **Ans.** Wave nature

23. **Ans.** 1590 J **Reason:** $\Delta W_{AB} = 0$ as V = constant.
$\therefore \Delta Q_{AB} = \Delta U_{AB} = 50$ J.
$U_A = 1500$ J. $\therefore U_B = (1500 + 50)$J = 1550 J.
$\Delta W_{BC} = \Delta U_{BC} = -40$ J.
$\therefore \Delta U_{BC} = 40$ J. $\therefore U_C = (1550 + 40)$J = 1590 J.

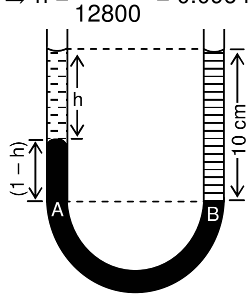
24. **Ans.** $\left(\frac{\pi}{6}\right)^{1/3} : 1$ **Reason:** $Q = \sigma A t\,(T^4 - T_0^4)$
If T, T_0 σ and t are same for both bodies then
$\frac{Q_{sphere}}{Q_{cube}} = \frac{A_{sphere}}{A_{cube}} = \frac{4\pi r^2}{6a^2}$ …… (i)
But according to problem, volume of sphere = Volume of cube $\Rightarrow \frac{4}{3}\pi r^3 = a^3 \Rightarrow a = \left(\frac{4}{3}\pi\right)^{1/3} r$
Substituting the value of a in eqn. (i) we get
$$\frac{Q_{sphere}}{Q_{cube}} = \frac{4\pi r^2}{6a^2} = \frac{4\pi r^2}{6\left\{\left(\frac{4}{3}\pi\right)^{1/3} r\right\}^2}$$
$$= \frac{4\pi r^2}{6\left(\frac{4}{3}\pi\right)^{2/3} r^2} = \left(\frac{\pi}{6}\right)^{1/3} : 1$$

25. **Ans.** 10°C **Reason:**According to Newton's law of cooling
According to Newton's law of cooling.
$\frac{\theta_1 - \theta_2}{t} = K\left[\frac{\theta_1+\theta_2}{2} - \theta_0\right]$
In the first case, $\frac{(60-50)}{10} = K\left[\frac{60+50}{2} - \theta_0\right]$
$1 = K[55-\theta_0]$ …… (i)
In the second case, $\frac{(50-42)}{10} = K\left[\frac{50+42}{2} - \theta_0\right]$
$0.8 = K\,(46 - \theta_0)$ …… (ii)
Dividing (i) by (ii), we get $\frac{1}{0.8} = \frac{55-\theta_0}{46-\theta_0}$
or $46 - \theta_0 = 44 - 0.8\,\theta_0 \Rightarrow \theta_0 = 10$°C

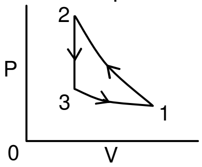
26. **Ans**. 450 m/s **Reason :** By the conservation of momentum
$0.05 \times \upsilon_0 = (5 + 0.05)\upsilon$ or $\upsilon = \frac{0.05\upsilon_0}{5.05} = \frac{\upsilon_0}{101}$
Kinetic energy of the interlocked body = work done against frictional force.
$\therefore \frac{1}{2}\times 5.05\times\left(\frac{\upsilon_0}{101}\right)^2 = 0.2 \times 5.05\, g \times 5$
or $\upsilon_0 = 101\sqrt{2\times9.8} = 447$ ms^{-1}

27. **Ans**. $\frac{\omega_0 l}{\sqrt{1+\frac{3m}{M}}}$ **Reason.** Using the law of conservation of angular momentum $I\omega_0 = (I + ml^2)\,\omega$ …………… (i)
Using the principle of conservation of energy
$\frac{1}{2}I\omega_0^2 = \frac{1}{2}(I + ml^2)\omega^2 + \frac{1}{2}m\upsilon'^2$
or $I\omega_0^2 = (I + ml^2)\frac{I^2\omega_0^2}{(I+ml^2)^2} + m\upsilon'^2$
$I\omega_0^2(I + ml^2) = I^2\omega_0^2 + m\upsilon'^2(I + ml)^2$
or $I\omega_0^2 ml^2 = m\upsilon'^2(I + ml)^2$
Putting $I = \frac{1}{3}Ml^2$, $\frac{1}{3}Ml^2\omega_0^2 l^2 = \upsilon'^2\left(\frac{1}{3}Ml^2 + ml^2\right)$
$\frac{1}{3}M\omega_0^2 l^2 = \upsilon'^2\left(m + \frac{M}{3}\right)$ or $M\omega_0^2\, l^2 = \upsilon'^2\,(3m + M)$
or $\upsilon' = \frac{\omega_0 l}{\sqrt{1+3m/M}}$

28. **Ans**. 9.6 cm **Reason.** Draw a horizontal line through the mercury-glycerin surface. This is a horizontal line in the same liquid at rest namely, mercury. Therefore, pressure at the points A and B must be the same.
Pressure at A = $p_0 + 0.1 \times (1.3 \times 1000) \times g$
p_0 = atmospheric pressure
Pressure at B = $p_0 + h \times 800 \times g$
$+ (0.1 - h) \times 13.6 \times 1000\, g$
$\therefore p_0 + 0.1 \times 1300 \times g$
$= p_0 + 800gh + 1360g - 13600 \times g \times h$
$\Rightarrow 130 = 800h + 1360 - 13600h$
$\Rightarrow h = \frac{1230}{12800} = 0.096$ m = 9.6 cm.



29. **Ans**. 3360 J **Reason.** A quick process is generally adiabatic and a slow process isothermal. Since 100 g of ice melts, heat given out by the system (gas in the cylinder) is equal to the required latent heat.



$\therefore \Delta Q = -100 \times 10^{-3} \times 336 \times 10^{3} = -3360$ J

$\Delta = 0$ (since it is a cyclic process)

But $\Delta Q = \Delta U + \Delta W$ (always)

$\therefore -3360 = \Delta W$

$\therefore$ work done on the system $= -\Delta W = 3360$ J

30. **Ans**. $\frac{1}{4\pi\epsilon_0}\frac{Q(R+r)}{R^2+r^2}$ **Reason.** $q_1 = 4\pi r^2 \sigma$ and $q_2 = 4\pi R^2\sigma$

$Q = q_1 + q_2 = 4\pi(r^2 + R^2)\sigma$ or $\sigma = \frac{Q}{4\pi(R^2 - r^2)}$

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r}+\frac{q_2}{R}\right] = \frac{1}{4\pi\varepsilon_0}\left[\frac{4\pi r^2\sigma}{r}+\frac{4\pi R^2\sigma}{R}\right]$$

$$\text{or } V = \frac{\sigma}{\varepsilon_0}(R+r) = \frac{1}{4\pi\varepsilon_0}\frac{Q(R+r)}{R^2+r^2}$$

CHEMISTRY

31. **Ans.** 16 g/mole **Reason:** In SCl_2, 71 parts of chlorine combine with 32 parts of sulphur

$\therefore$ 35.5 parts of chlorine combine with S = 16 parts

$\therefore$ Eq. mass of S in SCl_2 = 16.

32. **Ans.** 4.96 g **Reason:** According to the given reaction

$$2S_2O_3^{2-} \rightarrow S_4O_6^{2-} + 2e^-$$

$\therefore$ Eq. wt. of $Na_2S_2O_3 . 5 H_2O = \frac{\text{Mol. wt.}}{1} = \frac{248}{1} = 248$

100 cm^3 of 1 N sol require. $Na_2S_2O_3 . 5 H_2O$ = 248 g

$\therefore$ 100 cm^3 of 0.2 N of require

$Na_2S_2O_3 . 5H_2O = \frac{248 \times 0.2}{1000} \times 100 = 4.96$ g.

33. **Ans.** He^+

34. **Ans.** 25

35. **Ans.** II, III, I **Reason:** $H_2S_2O_6$ HO–S(=O)(=O)–S(=O)(=O)–OH

4π-bonds ; H_2SO_3 HO–S(=O)–OH 1π-bond;

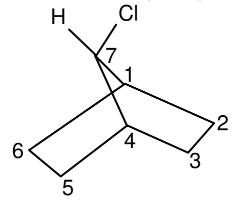
$H_2S_2O_5$ HO–S(=O)(=O)–S(=O)–OH 3π-bonds.

36. **Ans.** MgS

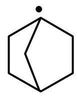
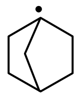
37. **Ans.** 4 **Reason:** Resonance is possible in sp^2 and sp-hybrid carbon atoms hence all sp and sp^2 hybrid carbon atoms are in the same plane. $\overset{5}{C}H_3 - \overset{4}{C} \equiv \overset{3}{C} - \overset{2}{C}H = \overset{1}{C}H_2$

C_3 and C_4 are sp hybridized so C_1, C_2, C_3 and C_4 are linear.

38. **Ans.** 7-chlorobicyclo [2, 2, 1] heptane **Reason:**



7-Chlorobicyclo [2, 2, 1] heptane

39. **Ans.**  **Reason:**  Bridgehead free radicals have pyramidal shape because due to steric strain, the carbon atom carrying the unpaired electron cannot assume a planar geometry.

40. **Ans.** 6 **Reason:** It is evident from figure that B occupies octahedral voids and thus, co-ordination number is six.

41. **Ans.** 5.49×10^7 C of electricity **Reason:**

$Al^{3+} + 3e \longrightarrow Al$

3F $\qquad$ 1 mole

$= 3 \times 96500$ C = 27 g

Thus, 27 g of Al require electricity = 3×96500 C

$\therefore$ 5.12 kg = 5120 g will require electricity

$= \frac{3 \times 96500}{27} \times 5120$ C $= 5.49 \times 10^7$ C

42. **Ans.** (i), (ii) and (iii) **Reason:** (i) High negative E° for $M^{n+} + ne^- \rightleftharpoons M$ means a high positive value for the reverse reaction, $M \rightleftharpoons M^{n+} + ne^-$. This means M is oxidized very easily. Hence, it is a good reducing agent.

(ii) Reduction potentials of halogens are in the order : $Cl_2 > Br_2 > I_2$. Thus, Cl_2 is reduced most easily and hence is the best oxidizing agent.

(iii) The size of the halide ions is in the order $Cl^- < Br^- < I^-$. Greater the size of the halide ion, more easily it can lose electrons and get oxidizied. Thus I^- ions can be oxidized most easily and hence have the greatest reducing power.

43. **Ans.** NaI < NaCl < BaO < CaO

44. **Ans.** CO_2 **Reason:** $AlF_3 \longrightarrow Al^{++} + F^-$

At anode $2F^- - 2e \longrightarrow F_2$

$Al_2O_3 + F_2 \longrightarrow AlF_3 + O_2$

$C + O_2 \longrightarrow CO_2$

45. **Ans.** Zinc oxide can be reduced by C.

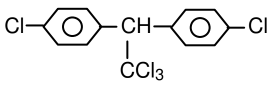
46. **Ans.** $FeSO_4$ **Reason:** $2FeSO_4 \longrightarrow Fe_2O_3 + SO_2 + SO_3$

$Fe_2O_3 + 6HCl \longrightarrow \underset{\text{Yellow soln.}}{2FeCl_3} + 3H_2O$

$FeCl_3 + CNS^- \longrightarrow \underset{\text{Blood red coloured soln.}}{Fe(CNS)_3} + 3Cl^-$

47. **Ans.** $[Ni(H_2O)_6]^{2+}$

48. **Ans.** $CH_3-\overset{O}{\overset{\|}{C}}-\overset{O}{\overset{\|}{C}}-CH_3$, $CH_3-\overset{O}{\overset{\|}{C}}-CHO$ and CHO—CHO

49. **Ans.** 

50. **Ans.** 2-Chloropropane-1, 3-diol **Reason:**

$$\overset{\delta+}{CH_2}=\overset{\delta-}{CH}\rightarrow CH_2OH\xrightarrow[\text{Anti-Mark addn.}]{\overset{\delta-\ \delta+}{HO-Cl}}\underset{\text{2-Chloropropane-1,3-diol}}{HOCH_2-CHCl-CH_2OH}$$

51. **Ans.** $NH_2\,NH_2$, OH^-

52. **Ans.** Phenolphthalein **Reason:** Only Phenolphthalein does not posses antiseptic properties.

53. **Ans.** conc. H_2SO_4

54. **Ans.** CO **Reason:** $K_4Fe(CN)_6 + 6H_2SO_4 + 6H_2O \longrightarrow 2K_2SO_4 + FeSO_4 + 3(NH_4)_2SO_4 + 6CO$.

55. **Ans.** H_3PO_2.

56. **Ans.** C_2H_4 **Reason:** Let the formula of the hydrocarbon be C_xH_y. The combustion of the hydrocarbon can be shown as :

$$C_xH_y + \left(x+\frac{y}{4}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}H_2O$$

$$10\text{mL} \qquad 10\left(x+\frac{y}{4}\right)\text{mL} \qquad 10x\ \text{mL}$$

The first reduction in volume after explosion

$$10+10\left(x+\frac{y}{4}\right)-10x = 20 \ = 10+\frac{10y}{4}=20$$

Thus, $y=\frac{10\times4}{10}=4$

Volume of carbon dioxide produced = 20 mL

Thus, $10x = 20$

$x=\frac{20}{10}=2$. Hence, the molecular formula of the hydrocarbon = C_2H_4.

57. **Ans.** 38.71 g **Reason:** $\therefore\ \Delta T = \frac{1000\times K_f\times w}{W\times m}$

$$9.3 = \frac{1000\times1.86\times50}{62\times W}$$

$\therefore$ W = 161.29

$\therefore$ Ice separated = 200 – 161.29 = 38.71 g

58. **Ans.** 64157 kcal **Reason:** ΔH required for heating = $(ms\ \Delta T)_{boiler} + (ms\ \Delta T)_{water}$

= 900 × 0.11 × 90 + 400 × 1 × 90= 44910 kcal

Since only 70% of heat given is used up to do so.

Actual heat required = $\frac{44910\times100}{70}$ = 64157 kcal

59. **Ans.** 7.5×10^{-4} M min^{-1} **Reason:**

$$k = \frac{2.030}{t}\log\frac{[A_0]}{[A]} = \frac{2.303}{40\,\text{min}}\log\frac{0.1}{0.005} = 0.075\ \text{min}^{-1}$$

Rate of reaction when concentration of x is 0.01 M

= k (X)

= 0.075 × 0.01 M min^{-1} = 7.5×10^{-4} M min^{-1}.

60. **Ans.** 345, 414. **Reason.** $2C(s) + 3H_2(g) \longrightarrow C_2H_6(g)$

$$\Delta_fH^\circ = \begin{bmatrix}2\times\Delta_{sub}H(C,s)\\3\times B.E.(H-H)\end{bmatrix} - \begin{bmatrix}B.E.\ (C-C)\\+6\times B.E.(C-H)\end{bmatrix}$$

-85 = 2× 718 + 3× 436 – (x + 6y)

x + 6y = 2829 ... (1)

Similarly, for $C_3H_8(g)$

2x + 8y = 4002 ... (2)

from eqs. (1) and (2)

x = 345 kJ/mol ; y = 414 kJ/mol.

MATHEMATICS

61. **Ans.**16 **Reason :** Let $E = \frac{(m^2-n^2)^2}{mn}$

$$\Rightarrow E = \frac{(m+n)^2(m-n)^2}{mn} \Rightarrow E = \frac{(2\tan A)^2(2\sin A)^2}{\tan^2 A-\sin^2 A}$$

$$\Rightarrow E = \frac{16\tan^2 A\sin^2 A}{\sin^2 A\left(\frac{1-\cos^2 A}{\cos^2 A}\right)}$$

$$\Rightarrow E = \frac{16\tan^2 A\sin^2 A}{\tan^2 A\sin^2 A} = 16.$$

62. **Ans.** 4. **Reason :** $s = \frac{13+14+15}{2} = 21$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{21.8.7.6} = 84$

Inradius = 84/21 = 4

63. **Ans.** ±1 **Reason :** We have $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = \frac{5\pi^2}{8}$

$$\Rightarrow (\tan^{-1}x+\cot^{-1}x)^2 - 2\tan^{-1}x\left(\frac{\pi}{2}-\tan^{-1}x\right) = \frac{5\pi^2}{8}$$

$$\Rightarrow \frac{\pi^2}{4} - 2.\frac{\pi}{2}\tan^{-1}x + 2(\tan^{-1}x)^2 = \frac{5\pi^2}{8}$$

$$\Rightarrow 2(\tan^{-1}x)^2 - \pi\tan^{-1}x - \frac{3\pi^2}{8} = 0$$

$$\Rightarrow \tan^{-1}x = -\frac{\pi}{4},\frac{3\pi}{4} \Rightarrow \tan^{-1}x = -\frac{\pi}{4} \Rightarrow x = -1.$$

64. **Ans.** (3, 7).

65. **Ans.** 0 **Reason** Put λ = 0 on both sides.

$$\begin{vmatrix}0 & -1 & 3\\1 & 0 & -4\\-3 & 4 & 0\end{vmatrix} = t. \Rightarrow -12+12 = t$$

66. **Ans.** $(0, \infty)$. **Reason:** We have, $f(x) = \log x - \frac{2x}{2+x}$

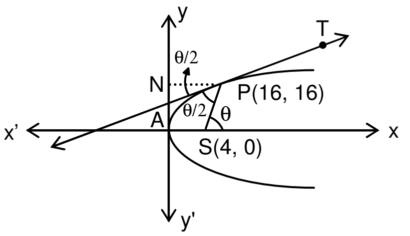
$$\Rightarrow f'(x) = \frac{1}{x} - \left[\frac{2(2+x)-2x}{(2+x)^2}\right] = \frac{(x+2)^2-4x}{x(2+x)^2}$$

$= \dfrac{x^2+4}{x(x+2)^2} = \dfrac{(x^2+4)x}{x^2(x+2)^2} > 0$, for x > 0

$\therefore$ f(x) is increasing for x > 0.

67. **Ans.** $\tan^{-1}(1/2)$ **Reason:** We know that PT bisects $\angle$NPS.

Let $\angle NPT = \angle TPS = \theta/2$. Then, $\angle PSX = \theta$.



$\Rightarrow \tan\theta = \dfrac{16-0}{16-4} \Rightarrow \tan\theta = \dfrac{4}{3}$

$\Rightarrow \dfrac{2\tan\theta/2}{1-\tan^2\theta/2} = \dfrac{4}{3} \Rightarrow 3\tan\dfrac{\theta}{2} = 2 - 2\tan^2\dfrac{\theta}{2}$

$\Rightarrow 2\tan^2\dfrac{\theta}{2} + 3\tan\dfrac{\theta}{2} - 2 = 0$

$\Rightarrow \left(2\tan\dfrac{\theta}{2} - 1\right)\left(\tan\dfrac{\theta}{2} + 2\right) = 0$

$\Rightarrow \tan\dfrac{\theta}{2} = \dfrac{1}{2}$ $\quad \left[\because \dfrac{\theta}{2} \text{ is acute}\right]$

$\Rightarrow \dfrac{\theta}{2} = \tan^{-1}\left(\dfrac{1}{2}\right) \Rightarrow \angle TPS = \tan^{-1}\left(\dfrac{1}{2}\right)$.

68. **Ans.** $\dfrac{\sqrt{155}}{2}$ sq. units **Reason:** Let the vertices be A (1, 2, 3), B (2, 5, –1) and C (–1, 1, 2), then area of Δ ABC = $\dfrac{1}{2}|\overrightarrow{AB} \times \overrightarrow{AC}|$

$= \dfrac{1}{2}|(\hat{i}+3\hat{j}-4\hat{k}) \times (-2\hat{i}-\hat{j}-\hat{k})| = \dfrac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -4 \\ -2 & -1 & -1 \end{vmatrix}$.

69. **Ans.** $\dfrac{x-1}{2} = \dfrac{y-1}{3} = \dfrac{z-1}{1}$ **Reason:** Line is $\perp$ to 2x + 3y + z + 5 = 0 means line is || to normal of the plane. D.N. of the normal are 2, 3, 1.

70. **Ans.** x + y + z = 0 **Reason** Any plane containing the line $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$ is a (x + 1) + b (y –3) + c (z + 2) = 0 … (1)

Where –3a + 2b + c = 0 … (2)

This passes through (0, 7, –7)

$\therefore$ a + 4b –5c = 0 … (3)

From (2) and (3), we get

$\dfrac{a}{-10-4} = \dfrac{b}{1-15} = \dfrac{c}{-12-2} \Rightarrow \dfrac{a}{1} = \dfrac{b}{1} = \dfrac{c}{1}$

Hence the required plane is x + 1 + y –3 + z + 2 = 0 i.e., x + y + z = 0.

71. **Ans.** –3/2 **Reason** $(\vec{a}+\vec{b}+\vec{c})^2 =$

$\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$

$= a^2 + b^2 + c^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$

$\Rightarrow (\vec{0}) = 2(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) + 1 + 1 + 1$

$\Rightarrow (\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -\dfrac{3}{2}$.

72. **Ans.** $F_2 \cup F_3 \cup F_4 \cup F_1$. **Reason :** Since every rectangle, rhombus and square is a parallelogram so $F_1 = F_2 \cup F_3 \cup F_4 \cup F_1$.

73. **Ans.** ω^2. $\left(\omega = -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i\right)$. **Reason :** $\dfrac{a+b\omega+c\omega^2}{c+a\omega+b\omega^2} = \dfrac{a\omega+b\omega^2+c\omega^3}{\omega(c+a\omega+b\omega^2)} = \dfrac{1}{\omega} = \omega^2$.

74. **Ans.** H.P. **Reason :** Here, $\dfrac{x-a}{b} + \dfrac{x-b}{a} = \dfrac{b}{x-a} + \dfrac{a}{x-b}$

$\Rightarrow \dfrac{(x-a)^2-b^2}{b(x-a)} + \dfrac{(x-b)^2-a^2}{a(x-b)} = 0$

$\Rightarrow (x-a-b)\left\{\dfrac{x-a+b}{b(x-a)} + \dfrac{x-b+a}{a(x-b)}\right\} = 0$

$\Rightarrow$ (x – a – b) {a(x – a) (x – b) + ab(x – b) + b(x – a) (x – b) + ab(x – a)} = 0

$\Rightarrow$ x(x – a – b) {a(x – a) + (x – b)} = 0

$\Rightarrow$ x =0, a+ b, $\dfrac{a^2+b^2}{a+b}$ so, x_1 = a+b, $x_2 = \dfrac{a^2+b^2}{a+b}$, x_3 = 0

Now, $x_1 - x_2 - x_3 = c \Rightarrow c = \dfrac{2ab}{a+b} \Rightarrow$ a, c, b are in H.P.

75. **Ans.** 50 **Reason:** We have, $\left[\dfrac{1}{2} + \dfrac{x}{100}\right] = \begin{cases} 0, & \text{if } 0 < x < 49 \\ 1, & \text{if } 50 \le x \le 99 \end{cases}$

Thus, $\left[\dfrac{1}{2}\right] + \left[\dfrac{1}{2} + \dfrac{1}{100}\right] + \left[\dfrac{1}{2} + \dfrac{2}{200}\right] + ... + \left[\dfrac{1}{2} + \dfrac{99}{100}\right]$ =50.

76. **Ans.** f(x) = log x **Reason** …

77. **Ans.** 12 **Reason:** Required limit

$= \lim_{x \to 0} \dfrac{2f'(x) - 6f'(2x) + 4f'(4x)}{2x} \left(\dfrac{0}{0} \text{form}\right)$

$= \lim_{x \to 0} \dfrac{2f''(x) - 12f''(2x) + 16f''(4x)}{2}$ = 3 f "(0) =12.

78. **Ans.** $\dfrac{y}{x}$. **Reason :** Given $x^p y^q = (x+y)^{p+q}$. Taking long on both sides, we get

$\Rightarrow$ p log x + q log y = (p + q) log (x + y)

Diff. both sides w.r.t. x., we get

$\dfrac{p}{x} + \dfrac{q}{y}\dfrac{dy}{dx} = (p+q)\left(\dfrac{1}{x+y}\right)\left(1 + \dfrac{dy}{dx}\right)$

$\Rightarrow \left(\dfrac{q}{y} - \dfrac{p+q}{x+y}\right)\dfrac{dy}{dx} = \dfrac{p+q}{x+y} - \dfrac{p}{x}$

$\Rightarrow \left(\dfrac{qx+qy-py-qy}{y(x+y)}\right)\dfrac{dy}{dx} = \dfrac{px+qx-px-py}{x(x+y)}$

$\Rightarrow \left(\dfrac{qx-py}{y(x+y)}\right)\dfrac{dy}{dx} = \dfrac{qx-py}{x(x+y)} \Rightarrow \dfrac{dy}{dx} = \dfrac{y}{x}$.

79. **Ans.** $\dfrac{x}{y}$. **Reason:** $u = e^{x^2+y^2}$

$\Rightarrow \dfrac{\partial u}{\partial x} = e^{x^2+y^2}(2x), \; \dfrac{\partial u}{\partial y} = e^{x^2+y^2}(2y)$

$\therefore \dfrac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \dfrac{2xe^{x^2+y^2}}{2ye^{x^2+y^2}} = \dfrac{x}{y}$.

80. **Ans.** $-\frac{\pi^2}{8}$. **Reason:** z = e^{xy^2} ; x = tcost; y = tsint, then

$\frac{dz}{dt} = \frac{\partial z}{\partial x}.\frac{dx}{dt} + \frac{\partial z}{\partial y}.\frac{dy}{dt}$

= e^{xy^2} . y^2 (–tsint + cost) + 2xy e^{xy^2} (tcost + sint)

= $t^2\sin^2 t\, e^{t^3}$ cost $\sin^2$t(–tsint + cost) + 2t^2sint cost e^{t^3} cost $\sin^2$t(tcost + sint)

$\left(\frac{dz}{dt}\right)_{at\ t=\frac{\pi}{2}} = \frac{\pi^2}{4}\left(-\frac{\pi}{2}\right) = -\frac{\pi^3}{8}$.

81. **Ans.** 4. **Reason:** The equation of the line is y – 3 = $\frac{3+2}{0-5}$ (x – 0), i.e., x + y – 3 = 0.

Now, slope of tangent to y = $\frac{c}{x+1}$ is ⇒ $\frac{dy}{dx} = \frac{-c}{(x+1)^2}$

Let the line touches the curve at (α, β)

$\therefore$ α + β – 3 = 0 and β = $\frac{c}{\alpha+1}$ …(1)

$\left.\frac{dy}{dx}\right|_{(\alpha,\beta)} = \frac{-c}{(\alpha+1)^2}$ = –1 …(2) ⇒ $\frac{c}{\left(\frac{c}{\beta}\right)^2}$

= 1 {using (1) or β^2 = c

⇒ β^2 = c⇒ $(3-\alpha)^2 = c = (\alpha+1)^2$ {using (1)}

⇒ 3 – α = ±(α + 1)⇒ 3 – α = α + 1

$\therefore$ α = 1

So, c = $(1 + 1)^2$ = 4. {using (2)}

82. **Ans.** 12.**Reason:** Since f(x) is decreasing in the interval (–2, –1), therefore, f(x) < 0 ⇒ $6x^2 + 18x + \lambda < 0$.
The value of λ should be such that the equation
$6x^2 + 18x + \lambda = 0$ has roots – 2 and –1
Therefore, (–2) (–1) = λ/6 ⇒ λ = 12.

83. **Ans.** f'(2) does not exist. **Reason:** In the interval [–1, 2], f'(x) = 6x + 12 > 0. Hence f(x) is increasing in [–1, 2].
Now, f(x) being a polynomial in x is continuous in
–1 ≤ x < 2. We check at x = 2.

$\lim_{h\to 0} f(2-h) = \lim_{h\to 0} 3(2-h)^2 + 12(2-h) - 1$

= 12 + 24 – 1 = 35

$\lim_{h\to 0} f(2+h) = \lim_{h\to 0} 37-(2+h)$ = 35.

Also, f(2) = $3(2)^2$ + 12(2) – 1 = 35.

$\therefore$ f(x) is continuous at x = 2 and hence in the interval [–1, 3].

Now, Lf'(2) = $\lim_{h\to 0} \frac{f(2-h)-f(2)}{-h}$

= $\lim_{h\to 0} \frac{3(2-h)^2+12(2-h)-1-35}{-h}$

= $\lim_{h\to 0} \frac{3h^2-24h}{-h}$ = 24.

Rf'(2) = $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = \lim_{h\to 0} \frac{37-(2+h)-35}{h}$ = –1.

84. **Ans.** $-\frac{\cos^4 x}{4} + c$ **Reason** : $\int \cos^3 x \sin x\, dx = -\int t^3 dt = -\frac{\cos^4 x}{4} + c$, where t = cos x.

85. **Ans.** $\frac{n\bar{x} - x_p + x'_p}{n}$ **Reason** Since the mean = $\bar{x}$, therefore sum of the n observations = n$\bar{x}$.
When x_p is replaced by x_p', then the new sum= n$\bar{x}$ -x_p + x_p'

So, new mean = $\frac{n\bar{x} - x_p + x'_p}{n}$

86. **Ans.** 4/7 **Reason** Let E_1 denote the event "getting face marked i", where, i = 1,2, …, 6
Then, P(E_i) = λi [Given]
Clearly, E_1, E_2, … E_6 are mutually exclusive and exhaustive events, therefore,
P($E_1 \cup E_2 \cup \ldots \cup E_6$) = P(S)

⇒ $\sum_{i=1}^{6} P(E_i) = 1 \Rightarrow \sum_{i=1}^{6} \lambda i = 1$ ⇒ 21λ = 1 ⇒ λ = 1/21.

$\therefore$ P (E_i) = i/21, i = 1, 2, …, 6.
Required probability = P ($E_2 \cup E_4 \cup E_6$)
= P(E_2) + P (E_4) + P (E_6)

$\frac{2}{21} + \frac{4}{21} + \frac{6}{21} = \frac{4}{7}$.

87. **Ans.** 1/969 **Reason** Let A, B, C, D denote events of getting a white ball in first, second, third and fourth draw respectively. Then Required probability
= P (A ∩ B ∩ C ∩ D)
= P (A) P (B/A) P (C/ A ∩ B) P (D/ A ∩ B ∩ C) … (1)
Now P (A) = Probability of drawing a white ball in first draw = 5/20 = 1/4.
When a white ball is drawn in the first draw there are 19 balls left in the bag, out of which 4 are white.
$\therefore$P (B/A) = 4/19
Since the ball drawn is not replaced, therefore after drawing a white ball in second draw there are 18 balls left in the bag, out of which 3 are white.
$\therefore$ P (C/ A ∩ B) = 3/18 = 1/6
After drawing a white ball in third draw there are 17 balls left in the bag, out of which 2 are white.
$\therefore$ P (D/ A ∩ B ∩ C) = 2/17
Hence, required probability = P (A ∩ B ∩ C ∩ D)
= P (A) P (B/ A) P (C/ A ∩ B) P (D/ A ∩ B ∩ C)

= $\frac{1}{4} \times \frac{4}{19} \times \frac{1}{6} \times \frac{2}{17} = \frac{1}{969}$.

88. **Ans.** $\left(0, \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)\right)$ **Reason** The equation of the biggest circle is $x^2 + y^2 = 1^2$
Clearly, it is Centered at O (0, 0) and has radius 1.
Let the radii of the other two circles be 1–r, 1 – 2r, where r > 0.
Thus, the equations of the concentric circles are:
$x^2 + y^2 = 1$ … (1)
$x^2 + y^2 = (1 - r)^2$ … (2)
$x^2 + y^2 = (1-2r)^2$ … (3)
Clearly, y = x + 1 cuts the circle (1) at (1, 0) and (0, 1).
This line will cut circles (2) and (3) in real and distinct points if $\left|\frac{1}{\sqrt{2}}\right| < 1 - r$ and $\left|\frac{1}{\sqrt{2}}\right| < 1 - 2r$

$\Rightarrow \frac{1}{\sqrt{2}} < 1 - r$ and $\frac{1}{\sqrt{2}} < 1 - 2r$

$\Rightarrow r < 1 - \frac{1}{\sqrt{2}}$ and $r < \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)$

$\Rightarrow r < \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right) \Rightarrow r \in \left(0, \frac{1}{2}\left(1 - \frac{1}{\sqrt{2}}\right)\right)$.

89. **Ans.** $\begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix} = 0$

Reason Let the given circles be $S_1 = 0$ and $S_2 = 0$. Let P, Q, R, S lie on the circles $S_3 = 0$.
Since Ax + By + C = 0 cuts the circle $S_1 = 0$ at P and Q. Therefore, Ax + By + C = 0 is the radical axis of $S_1 = 0$ and $S_3 = 0$.
Similarly, A'x + B'y + C' = 0 is the radical axis of $S_2 = 0$ and $S_3 = 0$.
The radical axis of $S_1 = 0$ and $S_2 = 0$ is
$S_1 - S_2 = 0$
i.e., x (a – a') + y (b – b') + c – c' = 0
Since the radical axis of three circle, taken in pairs, are concurrent. Therefore,
Ax + By + C = 0
A'x + B'y + C' = 0
and, x (a–a') + y (b –b') + c – c' = 0 are concurrent. Consequently, we have

$$\begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix} = 0$$

The equation of a family of circles passing through P and Q is

$x^2 + y^2 + ax + by + c + \lambda (Ax + By + C) = 0$ … (1)

Similarly, the equation of a family of circles passing through R and S is

$x^2 + y^2 + a'x + b'y + c' + \mu (A'x + B'y + C') = 0$ … (2)

If points P, Q, R and S are concyclic points, then equations (1) and (2) must represent the same circle for some for values of λ and μ.

$\therefore a + \lambda A = a' + \mu A'$

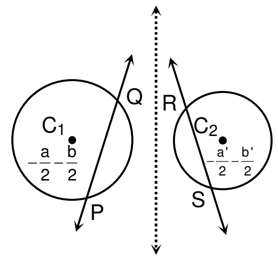
$b + \lambda B = b' + \mu B'$

$c + \lambda C = c' + \mu C'$

$\Rightarrow a - a' + \lambda A - \mu A' = 0$ … (3)

$b - b' + \lambda B - \mu B' = 0$ … (4)

$c - c' + \lambda C - \mu C' = 0$ … (5)



Eliminating λ and μ from equations (3), (4) and (5), we obtain

$$\Rightarrow \begin{vmatrix} a-a' & b-b' & c-c' \\ A & B & C \\ A' & B' & C' \end{vmatrix} = 0.$$

90. **Ans.** 7/6 sq. units **Reason** We have, $y = x^2 + x + 1$

$\Rightarrow \frac{dy}{dx} = 2x + 1 \Rightarrow \left(\frac{dy}{dx}\right)_{(1,\,3)} = 3$.

The equation of the tangent to $y = x^2 + x + 1$ at (1, 3) is $y - 3 = 3(x - 1) \Rightarrow y = 3x$.
The equation $y = x^2 + x + 1$ represents a parabola opening upward and having vertex at $\left(-\frac{1}{2}, \frac{3}{4}\right)$. Its graph is shown in figure. The area enclosed by x = –1, y = 0, $y = x^2 + x + 1$ and y = 3x is shaded in figure. We slice the shaded region into vertical strips.
Required area = Area of region OABC + Area of region OCD

$= \int_{-1}^{0} y\,dx + \int_{0}^{1} (y_1 - y_2)\,dx$

$= \int_{-1}^{0} (x^2 + x + 1)\,dx + \int_{0}^{1} (x^2 + x + 1 - 3x)\,dx$

$= \left[\frac{x^3}{3} + \frac{x^2}{2} + x\right]_{-1}^{0} + \left[\frac{x^3}{3} - x^2 + x\right]_{0}^{1}$

$= -\left(-\frac{1}{3} + \frac{1}{2} - 1\right) + \left(\frac{1}{3} - 1 + 1\right)$ = 7/6 sq. units.